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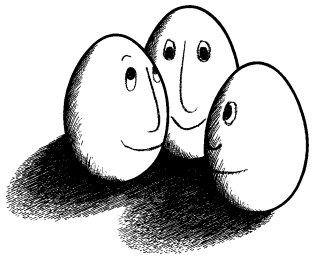

Computational Models of Learning in Astronomy

LS–8 Report 11

Martin Mühlenbrock

Dortmund, July 1994

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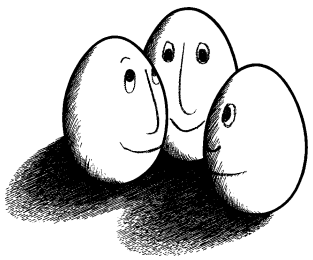

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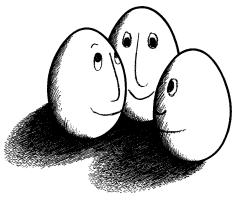

Computational Models of Learning in Astronomy*

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Dortmund, July 1994

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Abstract

Human learning appears to be heavily influenced by prior knowledge, yet the complex relationships between individual conceptions and their influence on the learning process are still subject to research. The computational representation of learning processes is assumed to yield a deeper insight into the interdependence of background knowledge and the product of learning.

Based on a cross age study on children's explanations of the day/night cycle conducted by S. Vosniadou and W. F. Brewer, children's conceptions of the celestial bodies and their conceptions of the appearance and disappearance of objects have been modeled within the knowledge representation system MOBAL. The formal and operational models help to specify the interconceptual relations and the conceptual development reconciling the culturally accepted scientific explanation of the day/night cycle with alternative conceptions.

1 Prior Knowledge in Human Learning

The earth is a huge sphere suspended in space. Is it really? How do you know? Well, the concept of the earth as being a huge sphere suspended in space is usually acquired sometime during childhood. But it is not the very first conception of the earth which is acquired at young age. Before children learn the scientifically accepted concepts of the earth, the sun, and the moon, they hold alternative conceptions of the celestial bodies, e.g., that the earth is flat and that it is supported by an infinite ground [Nussbaum, 1979, Mali and Howe, 1979, Sneider and Pulos, 1983, Vosniadou and Brewer, 1992]. And it seems plausible to assume that these alternative conceptions heavily influence further learning in astronomy [Vosniadou, 1991, Nussbaum, 1985].

There is a substantial body of evidence indicating the impact of prior knowledge on the learning process — a process leading to the construction of individual conceptual frameworks [Posner et al., 1982, Driver and Erickson, 1983, Driver, 1989, Duit, 1991]. These idiosyncratic knowledge structures are organized of concepts that take their meaning from the theories in which they are embedded [Murphy and Medin, 1985, Carey, 1985]. This perspective on human learning parallels the constructivist view on the way in which science itself proceeds [Novak, 1988, Nussbaum, 1989].

Learners are regarded as architects of their own learning through a process of equilibration between knowledge structures and new observations and information. Thereby, the learning process appears to be influenced by sequencing effects in two ways: On the one hand, the order in which information is available to the learner and observations are experienced by the learner may ease the formation of a particular concept. On the other hand, the availability of a particular concept may be prerequisite for acquiring another one. Hence, progress in understanding would be reflected by passing through a series of intermediate conceptions, with their sequence depending on the sequence of new information or observations.

Representing human learning on computer systems is assumed to advance the theory on conceptual change, i.e., how prior and new knowledge interfere within the process of human knowledge acquisition. Requiring explicitness, computational representations may help to specify and verify existing hypotheses on conceptual change, as well as they may suggest directions for further investigations by way of indicating the formation of new hypotheses [Strube et al., 1993]. In addition, computer systems facilitate not only the representation of individual conceptions, but also the simulation of conceptual *development*. [Simon and Kaplan, 1989]. Under this perspective, major interest lies in the investigation of the progressive evolution on the level of individual concepts and conceptual structures within specific, possibly knowledge intensive, real world domains.

The next section gives an overview of the knowledge representation system MOBAL, which has been used to represent human learning in the domain of observational astronomy. Starting from a cross age study on children's explanations of the day/night cycle [Vosniadou and Brewer, 1994], computational models have been developed to specify the precise relationships between children's individual concepts that are involved in explaining the alternation of day and night. Section 3 will outline these MOBAL models, followed by the specification of possible sequencing effects on their conceptual development in section 4.

2 Modeling Conceptual Structures

The constructivist view on human learning has been adapted to artificial knowledge representation systems, where it has been termed *Sloppy Modeling* [Morik, 1989]. The notion of sloppiness emphasizes the evolutionary character of computational models. A model is always incomplete and more or less adequate, since modeling is an incremental and yet infinite process. A knowledge representation system that is to assist in Sloppy Modeling would have to feature the extensive construction, inspection, and change of complex computational models.

The knowledge representation system MOBAL[1] constitutes a modeling environment designed to approximately meet the requirements of Sloppy Modeling [Morik et al., 1993, Sommer et al., 1993]. Design goals for the system have been to maintain incomplete and revisable knowledge, to check its consistency, and to enable the user to inspect the knowledge base. In particular, the inspection is eased by an immediate display of the consequences which the user's activities have.

Meaningful human learning relates to concepts and conceptual structures. In MOBAL, concepts and conceptual structures are represented within a restricted higher–order predicate logic. Concepts are referred to by means of predicates, i.e., predicates with single or multiple arity provide names for concepts. In addition, these predicates may have sorted arguments. For instance, consider the following unary predicate declaration

```
opaque/1: <object>
```

This declaration creates the name `opaque` for a concept of *opaqueness*, which applies to arguments of the sort `<object>`.

Concept membership is being expressed by facts. By specifying the members of a concept, facts represent the concept *extension*. Thus for instance the fact

```
opaque(hill).
```

states that a particular object, which is referred to by the constant term `hill`, is a member of the concept referred to as `opaque`. This might be read as *The hill is opaque*. At the same time, non–membership to a concept may be explicitly denoted by negated facts such as

```
not(opaque(cloud)).
```

which says that the object `cloud` is not a member of the concept referred to as `opaque`. This could mean that *The cloud is not opaque*.

Multi–ary predicates are to represent concepts that are rather *relational* than propositional in character. The following 4–ary predicate declaration

```
covers/4: <object>,<object>,<object>,<event>
```

creates the name `covers` for a concept of *covering*, which applies to quadruples of the sort `<object>,<object>,<object>,<event>`. A fact such as

```
covers(cloud,sun,me,event0).
```

[1] MOBAL has been developed at the German National Research Center for Computer Science (GMD).

states the membership of the quadruple `(cloud,sun,me,event0)` to the concept `covers`. The above fact specifies a relationship between the objects `cloud`, `sun`, `me` and the event `event0`. It might be read as *The cloud covers the sun for me in event0.*

A concept description (concept *intension*) consists of necessary and sufficient conditions, which are represented by rules. Necessary conditions are rules that contain the concept predicate as the only premise, e.g.,

```
covers(o1,o2,o3,e) -> disappears(o2,o3,e).
```

Note that `disappears` is also a concept predicate, which is to represent a concept of *disappearing*, and that `o1`, `o2`, `o3` and `e` are variables. This rule states a relationship between the concepts `covers` and `disappears`, which in turn denote relations themselves. It says that *covering* implies *disappearing*, or, more specifically, that *If one object covers another one from the viewpoint of a third object, then the second object disappears for the third one.*

Finally, sufficient conditions are rules that contain the concept predicate in the conclusion, such as

```
not(between(o1,o2,o3,s1)) & invisible(o2,o1,o3,s2) & state_seq(s1,e,s2)
& not(stationary(o1,e)) -> covers(o1,o2,o3,e).
```

This rule specifies that the concepts `between`, `invisible`, `state_seq`, and `stationary` imply the concept `covers`. This implication is further restricted by co–occurrences of variables in arguments of different concept predicates. As you can see, rule premises may be negated. The same applies to conclusions of rules.

The relationship between concepts is inferential in nature, i.e., MOBAL uses rules to infer facts by forward and backward chaining. Facts are assigned unique truth values, which could either be `true`, or `false`, or `both` for contradictory facts, or `unknown` for facts that are currently not inferable from the knowledge base. The truth values determine the inferences that can be made from the corresponding facts. The four valued interpretation of facts helps to maintain contradictory and incomplete knowledge. Contradictory knowledge will be displayed on an Agenda and is subject to a knowledge revision[2] and concept formation component of MOBAL.

Since rules link individual concept predicates, they represent conceptual structures. Conceptual structures might get rather complex, particularly when they are conceptions of real world phenomena, and so do their computational representations. Local concept changes might affect a number of other concepts, for concepts are closely related and changes may spread in conceptual structures. The system keeps track of all changes, be those either in the extension of a concept, represented by facts of the concept predicate, or in the intension of a concept, represented by rules with the concept predicate. Local changes are automatically and recursively propagated onto linked concept predicates.

Aside from maintaining changes, MOBAL offers several views to inspect the knowledge base, being quite helpful with complex conceptual structures. Among other things, the system organizes the domain predicates into interlinked sets of predicates. Two sets of predicates are linked if there are rules which have a predicate contained in one of the two sets as a premise and and a predicate from either set as a conclusion. A graphical

[2] The system resolves contradictions by *minimal base revisions*. For details refer to [Wrobel, 1993].

representation of the sets and their linkages, the so–called Predicate Topology, is generated by the system as an abstraction of the predicate intensions. It gives an overview of the conceptual structure that is currently being represented.

3 Explanations of the Day/Night Cycle

Observational astronomy appears to be a suitable domain for the investigation of conceptual change in children. Not only is it a relatively rich knowledge domain composed of a number of concepts with complex relations, but also does children's everyday experience provide them with sufficient information to develop an intuitive understanding of many of the phenomena that are part of the domain of scientific astronomy [Vosniadou and Brewer, 1992]. As we will see, scientific conceptions of astronomical phenomena differ from children's intuitive conceptions in some major respects. This makes the influence of scientific information on those intuitive conceptions a center of interest.

A number of psychological studies investigated children's conceptions of the celestial bodies such as the earth, the sun, and the moon and their conceptions of processes such as the day/night cycle and the waxing and waning of the moon. In a particular study, first, third, and fifth grade children were asked to explain the disappearance of the sun during the night, the disappearance of the stars during the day, and the alternation of day and night [Vosniadou and Brewer, 1994]. The study makes explicit the criteria used to identify children's ideas and provides information regarding the systematicity, consistency, and robustness of children's conceptions.

The children's explanations of the day/night cycle could be assigned to a limited number of conceptual frameworks, which have been modeled with MOBAL. The younger children retained conceptions that appeared to contain a certain concept of the earth, viz. that the earth is flat, being supported by an infinite ground [Vosniadou and Brewer, 1994]. Figures 1 and 2 exemplarily sketch two of these conceptions, showing graphical formalizations of the children's explanations. Model 1 explains the day/night cycle by virtue of clouds that cover the sun at night (see figure 1). In Model 3 the sun moves behind some hills at night and causes the day/night cycle in this way (see figure 2). Note that in Model 1 the sun seems to be stationary and the clouds move, whereas in Model 3 it is the sun that moves. In both models the earth is stationary, extending infinitely at least in downward direction.

Many of the older children that were probed in the study held the concept of an earth being a huge sphere suspended in space [Vosniadou and Brewer, 1994]. Some of these children think that the sun moves to the *other* side of the earth at night, this being the cause for the day/night cycle (Model 5 in figure 3). Just a few children appeared to have adopted an advanced conception similar to the scientific explanation of the day/night cycle (Model 9 in figure 4). This model contains — in contrast to the other three models presented here — the concept of an earth that is not stationary, but rotates on its axis and hence brings about the alternation of day and night.

Even though the children's ideas of the earth's *axis* have not been investigated in any major study on astronomy, none of the children would (probably) think that the earth's axis looks like some enormous dotted lines as figure 3 and 4 might suggest. These dotted lines rather refer to some characteristics of spatial reasoning within the formal

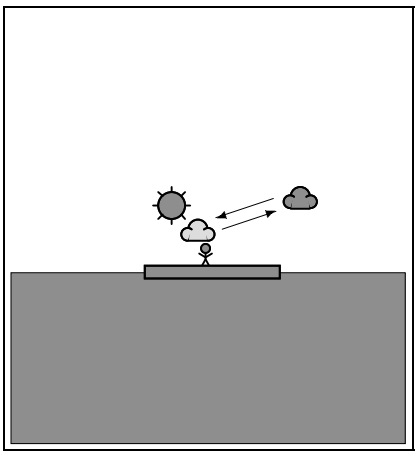

Figure 1: **Model 1** — *Clouds cover the sun.*

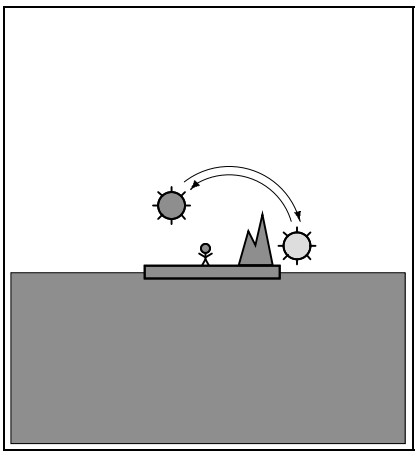

Figure 2: **Model 3** — *The sun moves behind hills.*

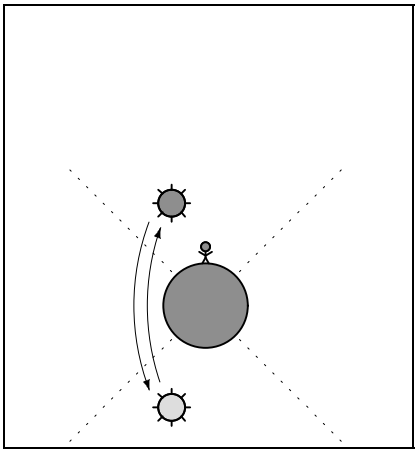

Figure 3: **Model 5** — *The sun moves to the other side of the earth.*

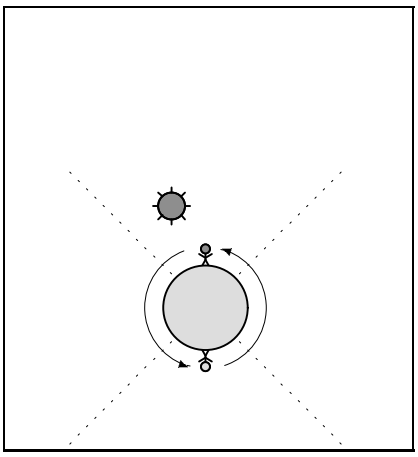

Figure 4: **Model 9** — *The earth rotates.* (scientific explanation of the day/night cycle)

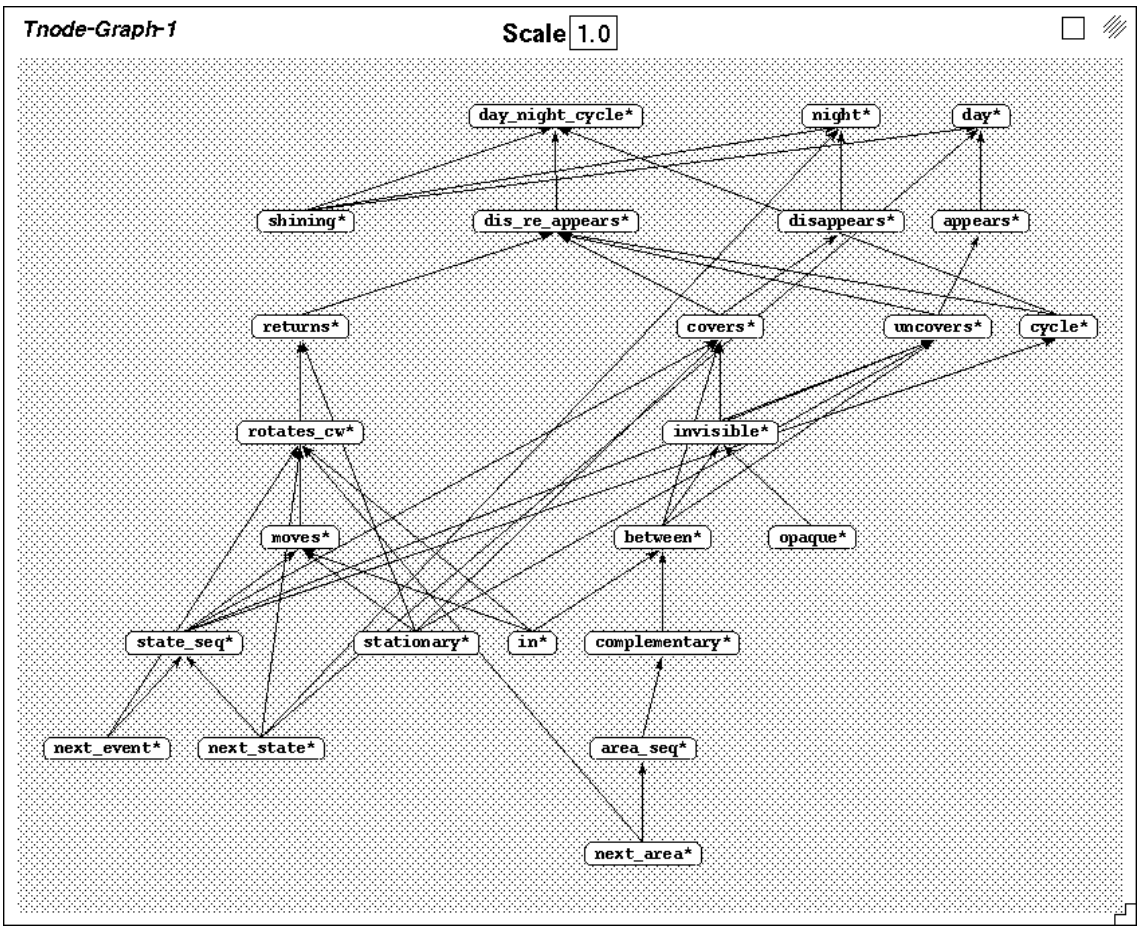

Figure 5: Predicate Topology of the computational models.

computational models that represent the children's conceptions. But instead of listing all the facts and rules that constitute the computational models of the day/night cycle, the easier–to–survey Predicate Topology will do better here. Figure 5 shows the Predicate Topology, which is an abstraction of all rules and depicts general intensional relationships between concept predicates (cf. section 2).

Within the Topology, you may recognize the predicates `opaque`, `covers`, `disappears`, `between`, `invisible`, `state_seq`, and `stationary`, which have been subject of the examples in section 2 to illustrate concept representation in MOBAL. These predicates and some others label the Topology nodes, with each node standing for one or more concept predicates. Hence not every declared predicate is visible. The Topology pictures the inferential relations between the concept predicates. It depicts how these concepts are embedded in the conceptual framework from that they take their meanings. Particularly, it shows how the concepts altogether define a concept of *day/night cycleness*, which is represented by the concept predicate `day_night_cycle` at the top of the Topology.

The Predicate Topology in figure 5 represents what is common to all computational models of the day/night cycle, viz. the concepts' intensions, which are identical for all of the four models presented here. Only the concepts' extensions, which are represented by

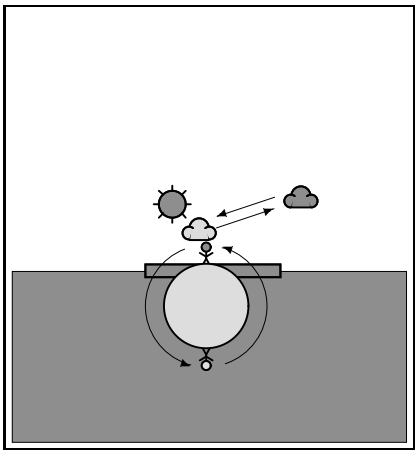

Figure 6: Sketched contradictions between Model 1 and Model 9.

facts (cf. section 2), vary from model to model and entail the different explanations of the day/night cycle. This means that the concepts' descriptions are rather general and apply to all models that explain the day/night cycle. Moreover, concepts such as *opaqueness*, *covering*, and *disappearing* not only apply to the domain of observational astronomy, but presumably to other domains as well.[3]

4 Conceptual Change in Observational Astronomy

Where do the specific differences between the individual explanations of the day/night cycle lie? This question can be answered with reference to the computational models. Figure 6, for instance, suggests where Model 1 and Model 9 may differ and where the conceptions they represent might contradict. Similarly, these contradictions occur with the computational models, yet on a more explicit and operational level. Figure 7 shows a MOBAL Agenda with contradictions between Model 1 and Model 9, gathered therein for further processing. Agenda entries either hint at contradictory facts, which call for a knowledge revision (cf. section 2), or signify violations of integrity constraints. An integrity constraint is a clause just like a rule, though its head can be empty or a disjunctive normal form expression [Morik et al., 1993]. Integrity constraints allow to state negative information without the explicit computation of all possible inferences. They are checked permanently by MOBAL or upon user request. Both kind of Agenda items will be illustrated below by way of example.

Contradictions between some model and Model 9 are of particular interest, since Model 9 represents the scientific explanation of the day/night cycle. These contradictions therefore hint at the information the alternative model should be exposed to in order to enhance its adequacy of explanation in regard to the day/night cycle. However, children only gradually get in touch with scientific information concerning the day/night cycle,

[3] For instance, the fact `covers(door,mother,me,event7)` implies `disappears(mother,me,event7)`.

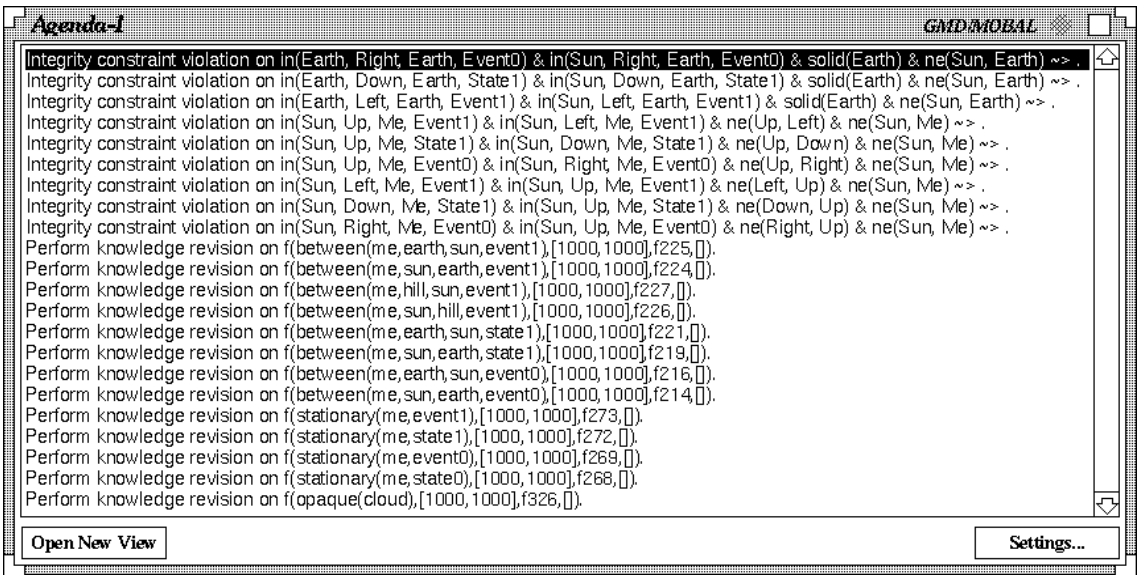

Figure 7: Agenda with contradictions between Model 1 and Model 9.

and this will change their conceptions only gradually, too. That means, the sequence of scientific information plays an important part, because different sequences of information will lead to different sequences of intermediate conceptions:

> *The evidence from a number of carefully conducted studies suggests that children's ideas within specific domains tend to follow certain trajectories. Moreover, although there is a variation at the individual level and there may be specific cultural influences to be considered, the general picture is that there is much in common in the conceptual trajectories for children from different backgrounds and from different countries.* [Driver, 1989, p. 488]

Such a trajectory, which is suggested by the computational models, might be construed by Model 1, Model 3, Model 5, and Model 9 (cf. figures 1 to 4). How can this sequence of models be characterized? Informally, the earth at first extends infinitely downward (Model 1 and Model 3), but then gets conceptualized as being spatially bounded in all directions (Model 5 and Model 9). The concept of an earth being entirely surrounded by space seems to be necessary to construct the concept of an earth that is not stationary (Model 9). The same conception is likely to be necessary to desist from the idea of some clouds covering the sun (Model 1), passing the idea of the sun hiding behind some hills (Model 3), to eventually attain the concept of a sun that hides on the other side of the earth (Model 5). But note that though the sun concept of being stationary (Model 1) is altered to being unstationary in Model 3 and Model 5, it is reverted to the initial conception in the scientific explanation of the day/night cycle (Model 9).

What might have been the information that led to this sequence of models? A sequence of information that is proposed by the computational models might be the following. The first model, Model 1, contains the information that the clouds are opaque and cover the sun to make it night on earth. The contradicting information or observation that the clouds are not opaque, represented by the fact

```
not(opaque(cloud)).
```

would reject the *covering* explanation of the day/night cycle. In figure 7, this contradiction is being indicated by the last Agenda item. In MOBAL, facts are interpreted by *evidence points*, which in principle are continuous truth values in a two–dimensional evidence space [Morik et al., 1993]. In particular, a pair of integers, each between 0 and 1000, represents positive and negative evidence. The symbolic truth values that have been introduced in section 2 are the corner points of this evidence space: `true` corresponds to `[1000,0]`, `false` to `[0,1000]`, `both` to `[1000,1000]`, and `unknown` to `[0,0]`. According to the last Agenda item in figure 7, the fact `opaque(cloud)` is assigned the evidence point `[1000,1000]`. Hence, this represents a contradiction between Model 1 and Model 9 in regard to the *opaqueness* of the clouds.[4]

Although the new information on clouds retracts Model 1, Model 3 constitutes a suitable explanation of the day/night cycle, since the hills are still being conceptualized as opaque (cf. section 2). However, the information that at night the sun is on the other side of the earth contradicts Model 3. This information may be expressed by

```
between(earth,sun,me,state1).
```

which partially corresponds to the facts

```
in(me,up,earth,state1).
in(sun,down,earth,state1).
```

On the contrary, Model 3 states with

```
in(earth,down,earth,state1).
```

that the earth itself is on the other side of the earth — meaning that it extends infinitely downward. Furthermore, Model 3 says that if an object is infinite and solid there may be no other object in the occupied area, which is represented by the integrity constraint[5]

```
in(o1,a,o1,t) & in(o2,a,o1,t) & solid(o1) & ne(o2,o1) ~>.
```

Since this integrity constraint is violated by the above facts, an Agenda entry is generated by the system (see the second Agenda item in figure 7).

Finally, Model 5 would be contradicted by the facts

```
in(sun,left,me,event0).
in(sun,right,me,event1).
```

which represent the information or observation that the sun sets in a direction opposite to the one where it rises. The additional fact

```
not(stationary(earth,event0)).
```

which might be read as *The earth is not stationary in event0*, may then lead to the acquisition of Model 9, which explains the day/night cycle by virtue of the daily rotation of the earth.

[4] In addition, the Agenda item is labeled with the index of the fact in the knowledge base.

[5] The predicate `ne` is a built–in predicate to express *unequalness* and the empty conclusion is to be read as 'fail' or 'false' [Morik et al., 1993].

5 Concluding Remarks

Complying with Sloppy Modeling, the computational models are to be developed further to improve their adequacy concerning the children's explanations. Methodologically this means, on the one hand, to conduct experiments with the computational models and to compare their answers to those the children gave in the underlying psychological study. On the other hand, it seems necessary to empirically validate those concepts that partly had to be postulated within the computational models in order to obtain explicit representations.

As mentioned in sections 2 and 3, the MOBAL formalism renders possible to state inferential relations between relational concepts and hence allows for rather general concept descriptions. For instance, essential to explanations of the day/night cycle are concepts of the *appearance*, *disappearance*, and *reappearance* of objects, as can be seen in the Predicate Topology in figure 5. It seems reasonable to assume that these concepts are rather acquired with everyday phenomena and are then applied in the construction of explanations of the alternation of day and night [Vosniadou and Brewer, 1994, Mühlenbrock, 1994]. This would account for the simplifying assumption that conceptual change in observational astronomy only consists of changing concept extensions, leaving concept intensions unchanged. However, this in turn raises the question of how the concept intensions are acquired in daily situations.

In addition, the computational models merely explain the robustness of particular concepts in children's explanations. On the one hand, the computational models do not account for the difference between received information and experienced observations. For instance, children's conceptions seemed to be persistently constrained by the presupposition that the earth is flat, as it in fact appears to be [Vosniadou and Brewer, 1992]. On the other hand, there seem to be concepts whose intensions are not likely to be acquired in everyday situations, such as the gravity concept. Hence the need to first construct an intensional concept description would account for sequencing effects that involve concept robustness.

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